Math 131B-1: Homework 4

Due: February 3, 2014

1. Read Apostol Sections 4.8-9, 4.11-13, 4.15-17, 4.19-20. [Most of these are short.]
2. Do problems 4.21, 4.25, 4.28, 4.33, 4.38, 4.39 in Apostol.
3. We say that a subset S of a metric space M is *dense* if every open set in M contains a point of S.
   - Prove that if S is dense in M, every point of M is the limit of a sequence of points in S.
   - Prove that if $f : (M, d_M) \to (T, d_T)$ and $g : (M, d_M) \to (T, d_T)$ are two continuous functions from M to a metric space (T, d_T), and $f(s) = g(s)$ for all $s \in S$, then $f = g$ on M.
4. Let $f : X \to \mathbb{R}^n$ be a function such that $f(x) = (f_1(x), \cdots, f_n(x))$. Show that f is continuous if and only if each function $f_i : X \to \mathbb{R}$ is continuous.